A Comparison of a Mean Field Theoretic Approach to Ferromagnetism with Experimental Results

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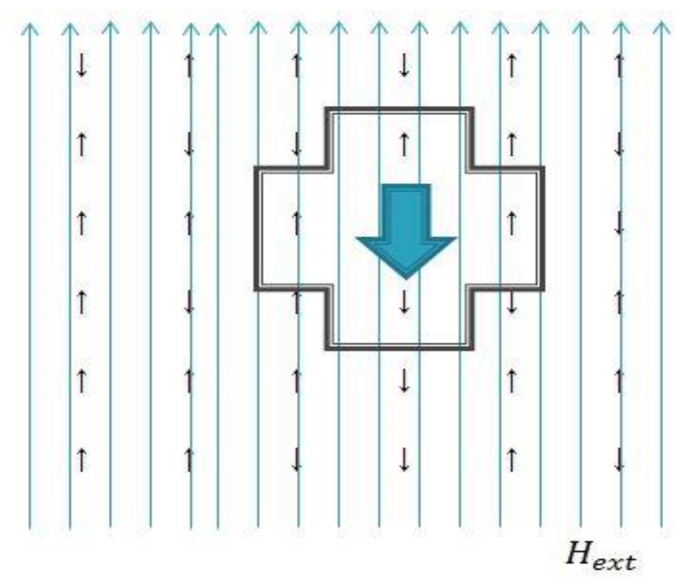
Abstract

Often, trying to describe how atomic interactions in a structure come to manifest themselves macroscopically is tedious if not impossible when large numbers of molecules or atoms are being considered. One way to more easily approximate the expected behavior of a large system of particles, such as magnetic dipoles, is to consider how the "mean field" produced by neighboring particles near a single particle affects it and observing its behavior. In magnetic systems, there is an electromagnetic exchange between individual dipoles that extends only negligibly beyond the other dipoles immediately adjacent to them. By "tagging" an individual dipole in a tetragonal crystal lattice of ferromagnetic dipoles and then "freezing" the dipoles immediately near it, the mean field produced by the frozen dipoles can be calculated, and the net effect of the mean field on the tagged dipole can be seen. In the case of ferromagnetism (and all magnetic systems), the magnetization and orientations of dipoles are dependent on temperature. To see this, several magnets were cooled with liquid nitrogen and allowed to warm back up to room temperature while the strength of their magnetic fields were measured at a constant distance. The observed field strengths are then compared with theoretical results produced from the mean field approximation.



Theoretical Background

- To make a mean field approximation of a large ferromagnetic system, we consider only a single dipole within a material and the interactions it has with the other dipoles immediately adjacent to it.
- After calculating how the dipole responds to the effective magnetic field produced by the surrounding dipoles, we generalize it to the system as a whole.

- The effective magnetic field produced by the neighboring dipoles is given by

$$H_{eff} = \frac{nJ}{\mu}\langle s\rangle$$

- If there is no external magnetic field, the energy of the single dipole being observed is

$$E_i = -H_{eff}\mu s_i$$

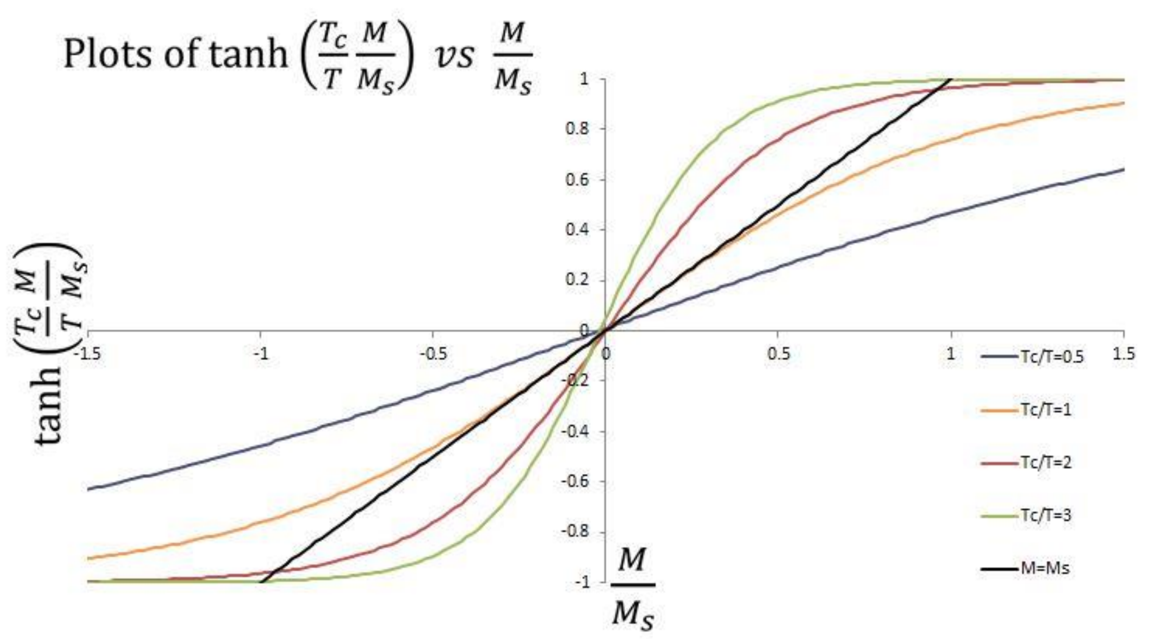
- The average spin alignment of the tagged dipole due its interaction with the surrounding dipoles is given by

$$\langle s\rangle = \tanh\left(\frac{\mu H_{eff}}{kT}\right) = \tanh\left(\frac{\mu\left(\frac{nJ}{\mu}\langle s\rangle\right)}{kT}\right) = \tanh\left(\frac{nJ\langle s\rangle}{kT}\right)$$

$$\Downarrow$$

$$\frac{M}{M_s} = \tanh\left(\frac{nJ}{kT}\frac{M}{M_s}\right) = \frac{M}{M_s} = \tanh\left(\frac{T_c}{T}\frac{M}{M_s}\right)$$

- This equation is transcendental and can be solved graphically to find solutions. Several sample solutions are given to the right.



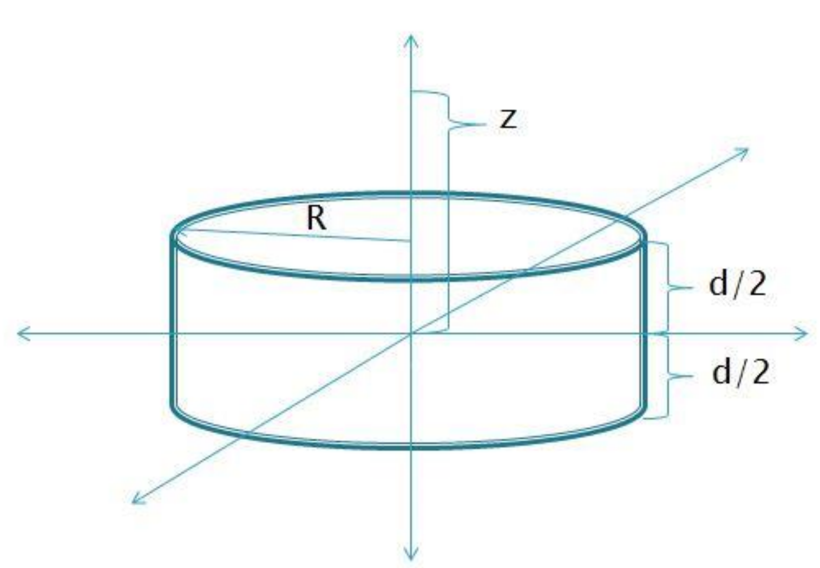
Constants, Terms and Definitions

- k, Boltzmann's Constant. $k = 1.381 \times 10^{-23}$ Joules/Kelvin.
- J, a coupling constant to describe the interaction between two dipoles, has units of Joules.
- μ, the magnetic dipole moment, has units of Joules/Tesla.
- s_i, the tagged dipole being observed
- $\langle s\rangle$, the average spin alignment of the neighboring dipoles.
- M, the magnetization of a system given by $M = n\mu\langle s\rangle$ for n dipoles., has units are Amps/meter
- M_s, the saturation magnetization of a system where all dipoles are perfectly aligned. $M_s = n\mu$ for n dipoles , has units of Amps/meter.
- T, the absolute temperature of a system, given in Kelvin.
- T_c, the Curie temperature. Defined as $T_c = \frac{nJ}{k}$ Kelvin.

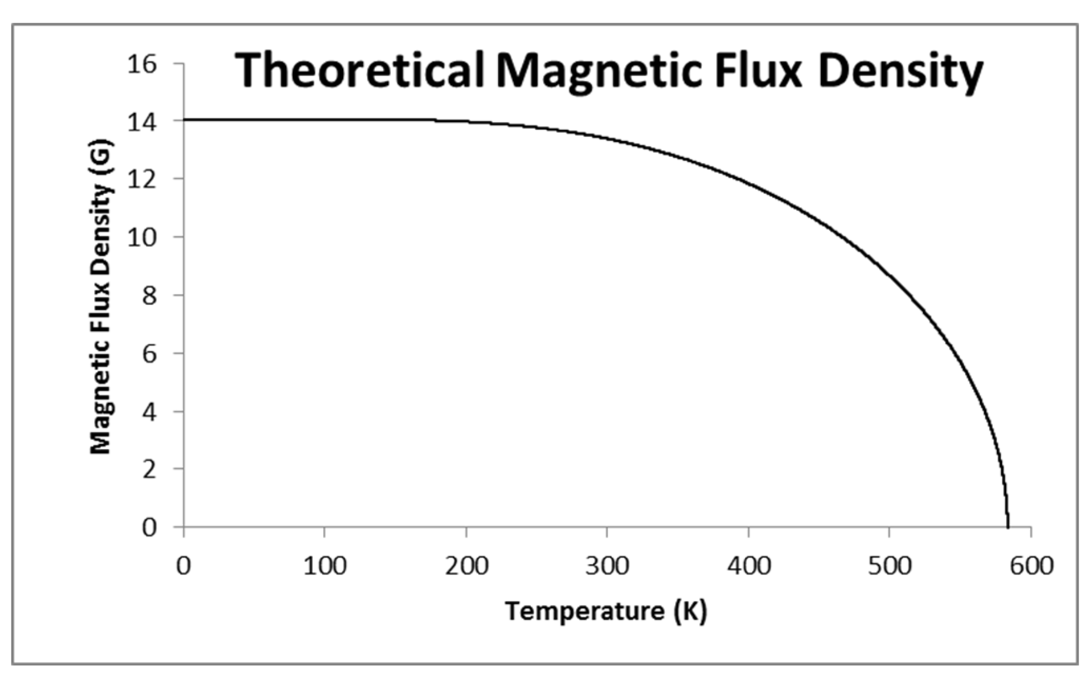
Predictions for the Magnetic Field for a Real Magnet

$$|H_z| = \frac{Md}{2}\left[\frac{\left(\frac{z}{d}-\frac{1}{2}\right)}{\sqrt{\frac{R^2}{d}+\left(\frac{z}{d}-\frac{1}{2}\right)^2}} - \frac{\left(\frac{z}{d}+\frac{1}{2}\right)}{\sqrt{\frac{R^2}{d}+\left(\frac{z}{d}+\frac{1}{2}\right)^2}}\right]$$

Where R= the radius of the magnet
d=the height of the magnet
z= the point along the z axis at which we want to measure the magnetic field

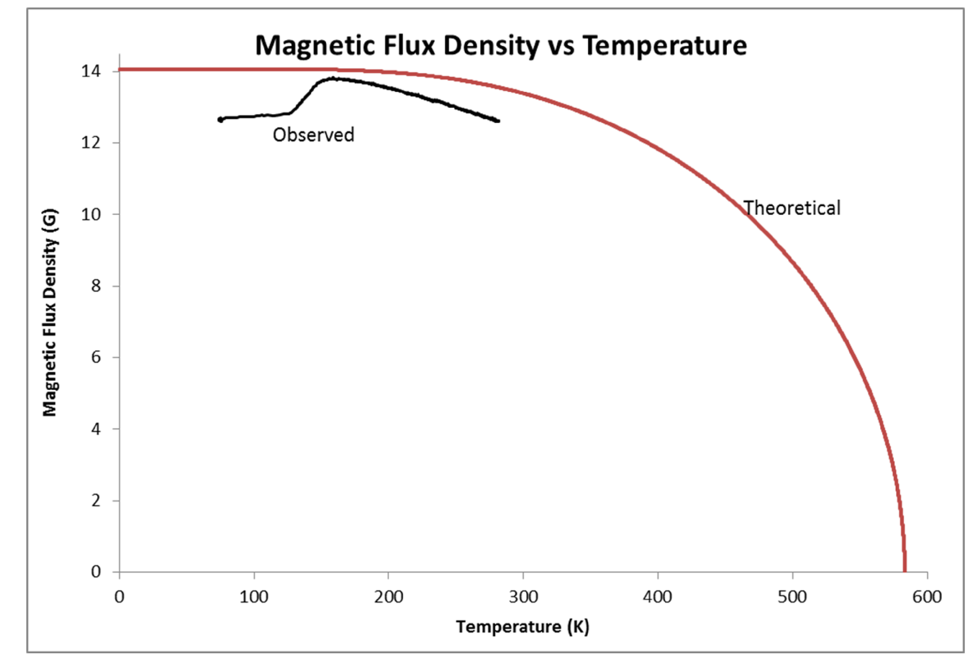


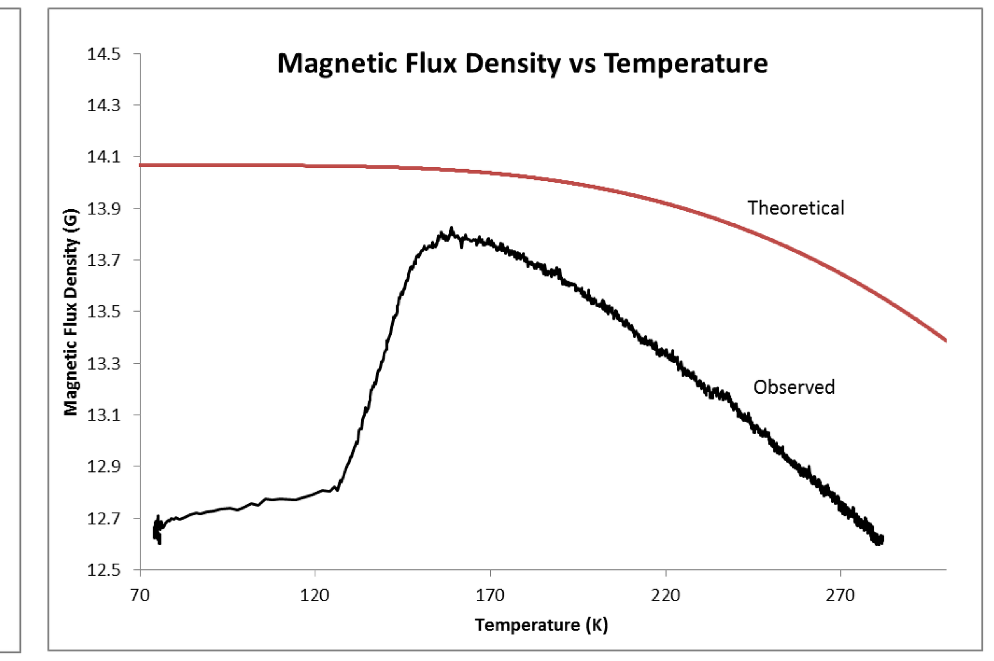
- The solutions to this curve are used to predict the magnetization in a real system as a function of temperature.
- This experiment was performed by taking measurements along the z axis of a disc shaped magnet. The magnetic field is given by the equation to the left.
- For a 1 inch diameter, 3/8 inch thick disc magnet, the theoretical magnetic field 9.3 cm above the disc is given below in units of Gauss.

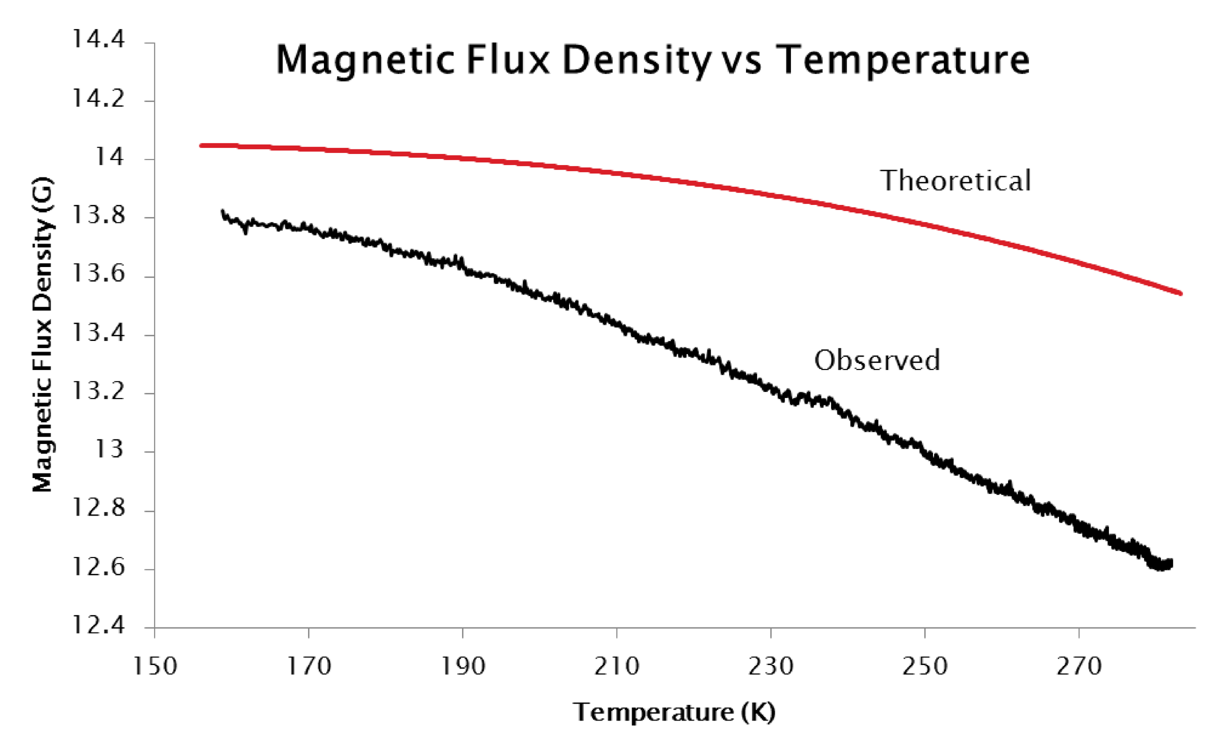


Observations in a $Nd_2Fe_{14}B$ Ferromagnet

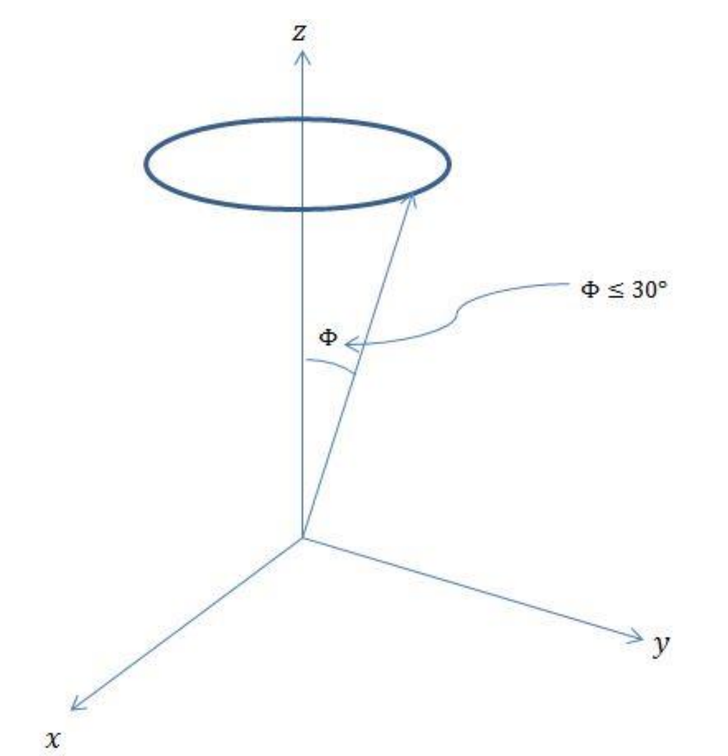
- The magnet was immersed in liquid nitrogen at 77K and allowed to warm back up to room temperature.
- The observed magnetic field is given by the two graphs to the right and compared with theoretical predictions.
- The magnetic field has a similar shape to the theoretical curve above 158.81K. Below 158.81 the magnet experienced a spin reorientation transition (SRT).
- Since MFT makes no account of the SRT, only data above the transition temperature was considered, as shown below.







- The Spin Reorientation Transition (SRT) was observed to occur at 158.81K and below.
- The SRT is due to a change within the material that effectively allows the magnetic dipoles to orient themselves anywhere within the cone shown to the right.
- Literature suggests that the SRT shouldn't cause the magnetic field to drop by more than 14%.
- The minimum magnetic field observed below the SRT is 8.79% lower than at the transition temperature, within the expected range.



Conclusions

- Theoretical predictions are an average of 4.35% higher than observed data.
- There was found to be a 4.12% error in the measurement of the magnetic field.
- At low temperatures, it was found that the theoretical curve was within the region of error.
- The accuracy of the MFT diminishes as the temperature of the system warms to room temperature.
- MFT doesn't take into account the SRT , and is unreliable for the temperature range over which the material experienced the SRT.